

B.E. / B.Tech. Mechanical Engineering (Model Curriculum) Semester-III
BSC202 - Applied Mathematics-III (PDE, Probability & Statistics)

P. Pages : 2

Time : Three Hours



GUG/S/25/14056

Max. Marks : 80

- Notes :
1. All questions carry equal marks.
 2. All questions are compulsory.
 3. Non programmable calculate is permitted.

1. a) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. 8

b) Solve $(D^3 - 3D' - 2D'^3)z = \cos(x + 2y) - e^y(3 + 2x)$ 8

OR

2. a) Show that $u = e^x(x \cos y - y \sin y)$ is harmonic and find v such that $f(z) = u + iv$ is an analytic function. 8

b) Show that $u = 2x - x^3 + 3xy^2$ is a harmonic function. 8
Find the harmonic conjugate and the corresponding analytic function $f(z) = u + iv$

3. a) An urn holds 5 white and 3 black marbles. If two marbles are drawn at random without replacement and X denotes the number of white marbles. 8
i) Find the probability function and
ii) The distribution function.

b) Let X be random variable with density function 8
$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{find } E(X) \text{ and } \text{Var}(X)$$

OR

4. a) A random variable X has the density function given by 8
$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the coefficient of skewness and kurtosis.

b) Find the distribution function for r.v. X whose density 8
Function is $f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$
Hence find $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$ and $P(X > 1)$.

5. a) Solve the differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$ given $X(0) = 1, x'(0) = 0$ by matrix method. 8

- b) Use Sylvester's theorem to verify that $\sin^2 A + \cos^2 A = 1$, Where $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. 8

OR

6. a) Verify Cayley Hamiltonian theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ And find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ As a linear polynomial in A. 8

- b) Find the modal matrix B corresponding to the matrix $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ and verify that $B^{-1}AB$ is a diagonal form. 8

7. a) Obtain the Fourier series for $f(x) = x - x^2$ in the interval $-1 < X < 1$ 8

- b) Give that $f(x) = x + x^2$ for $-\pi < x < \pi$ find the Fourier Expression of $f(x)$. 8

OR

8. a) Find L. T. of $\frac{e^{-at} - e^{-bt}}{t}$, hence evaluate $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}$ 8

- b) Find $L^{-1} \left\{ \frac{s}{s^4 + 4a^4} \right\}$ 8

9. a) Find by Newton's Rapson method, the real root of $3x - \cos x - 1 = 0$ 8

- b) Using Regula False method, find the roots of the following Equations correct to third decimal place $x \log_{10} x - 1.2 = 0$. 8

OR

10. a) Solve the following system of equations by Crout's method $x + y + z = 1, 3x + y - 3z = 5, x - 2y - 5z = 10$ 8

- b) Solve by Gauss-Seidel method, $x + 7y - 3z = -22, 5x - 2y + 3z = 18, 2x - y + 6z = 22$ 8
